# Why learn linear algebra?

## 1) Linear Programming

Uses matrices to encode movements (rotations, scalings, translations, etc.)

# 21 Search Algorithms

Google's PageRank algorithm involves finding eigenvectors for a particular matrix (Perron-Frobenius theory)

# 3) Signal Analysis

Take a "continuous" signal and "discretize" it. If there are only finitely many components, can use the Fourier Transform, represented as a matrix, to analyze the signal

# 4) Quantum Mechanics

All measurement and time-evolution operators are linear (though sometimes on an infinite-dimensional space)

# How do we know QM is linear?

- Experiments say so

- Theory gives the following consequences of nonlinear QM: Time travel, faster-than-light communication (Aronson)

# 5) Quantum Computation

Feynman's idea: construct a computer running on QM principles.

Gates become linear operators (matrices in finite dimensions)

Theoretically, a working quantum computer can crack the RSA algorithm

in polynomial time.

# 6) Image Recognition

Compressed sensing uses techniques on sparse (few nonzero entries) matrices to sharpen grainy images

# 7) Relativity

Einstein's Special Relativity makes use of transformations on Minkowski space (four-dimensional) represented by matrices

Which of these topics will you see in this course?

Maybe 7.

Maybe not.

# Mathematical Background & Techniques of Proof

## Sets

What is a set?

No definition, but for example, trees, cars, natural numbers, solutions to a particular differential equation

Consider all sets that do not contain themselves as a member

(e.g. the set of all trees is not a tree).

This is not a set!

The members of a set are called its elements.

Trees are the elements in the set of all trees.

Paul Halmos:
Naive Set Theory

# Subsets

If S is a set,

a Subset T of S

is a collection of

some of the elements

of S.

If S = natural numbers,

T could be all even

numbers.

# The Empty Set

The empty set is a set with no elements, like $\{x \mid x \neq x\}$.

Notation for empty set: either $\emptyset$ or $\{\emptyset\}$.

The empty set is a subset of all other sets.